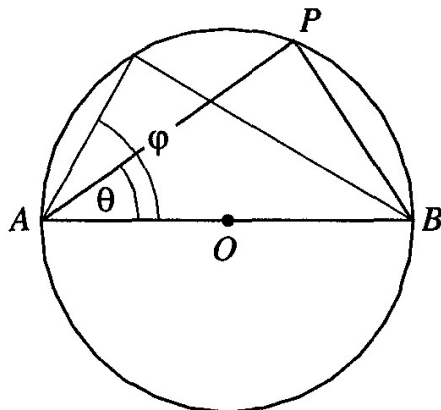


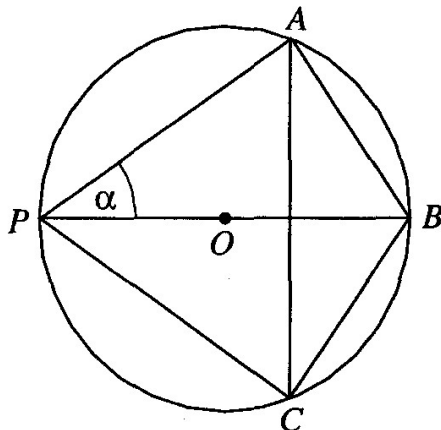
3. The diagram below shows a circle of diameter 1, and two acute angles θ and φ :



How does the diagram suggest that if $\varphi > \theta$, then $\sin \varphi > \sin \theta$?

4. We know from geometry that a circle may be drawn through the three vertices of any triangle. Find the radius of such a circle if the sides of the triangle are 6, 8, and 10.
5. Starting with an acute triangle, we can draw its circumscribed circle (the circle that passes through its three vertices). If α is any one of the angles of the triangle, show that the ratio $a : \sin \alpha$ is equal to the diameter of the circle.
6. Use Exercise 5 to show that if α, β, γ are three angles of an acute triangle, and $a, b,$ and c are the sides opposite them respectively, then
- $$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}.$$

7. The diagram below shows a circle with center O , and chords AB and AC :



Arc AC is double arc AB . Diameter BP , chord AP and chord CP are drawn in, and $BP = 1$ (the diameter of the circle has unit length). If angle APB measures α degrees, use this diagram to show that $\sin 2\alpha < 2 \sin \alpha$.

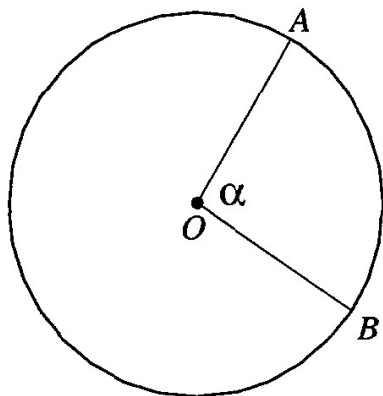
You may need the theorem known as the *triangle inequality*: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

8. In a circle of diameter 10 units, how long is a chord intercepted by an inscribed angle of 60 degrees?
9. In a circle of diameter 10 units, how long is a chord intercepted by a central angle of 60 degrees?
10. Find the length of a side of a square inscribed in a circle of diameter 10 units.
11. If you knew the exact numerical value of $\sin 36^\circ$, how could you calculate the side of a regular pentagon inscribed in a circle of diameter 10?

Appendix – Review of Geometry

I. Measuring arcs

One natural way to measure an arc of a circle is to ask what portion of its circle the arc covers. We can look at the arc from the point of view of the center of the circle, and draw the central angle that cuts off the arc:



If central angle AOB measures α degrees, then we say that arc \widehat{AB} measures α degrees as well.